**พิสูจน์เอกลักษณ์ตรีโกณมิติ ม.5 เทอม 1**

**P.7**

1. L.S. $= \sin A\left(\frac{\cos A}{\sin A}\right)$

$= \cos A$

$=$ R.S.

2. L.S. $= \cos A\left(\frac{\sin A}{\cos A}\right)$

$= \sin A$

$=$ R.S.

3. L.S. $= \left(\frac{\cos A}{\sin A}\right)\left(\frac{1}{\cos A}\right)$

$= \frac{1}{\sin A}$

$= \operatorname{cosec} A$

$=$ R.S.

4. L.S. $= \sin A\left(\frac{1}{\cos A}\right)$

$= \frac{\sin A}{\cos A}$

$= \tan A$

$=$ R.S.

5. L.S. $= \cos A\left(\frac{1}{\sin A}\right)$

$= \frac{\cos A}{\sin A}$

$= \cot A$

$=$ R.S.

6. L.S. $= \left(\frac{\cos A}{\sin A}\right)\left(\frac{1}{\cos A}\right)\sin A$

$= 1$

$=$ R.S.

7. L.S. $= \left(\sin^2 A\right)\left(\frac{1}{\sin^2 A}\right)$

$= 1$

$=$ R.S.

8. L.S. $= \left(\cos^2 A\right)\left(\frac{1}{\cos^2 A}\right)$

$= 1$

$=$ R.S.

9. L.S. $= \left(\frac{\cos^2 A}{\sin^2 A}\right)\left(\sin^2 A\right)$

$= \cos^2 A$

$=$ R.S.

10. L.S. $= \left(\sin^2\theta\right)\left(\frac{1}{\cos^2\theta}\right)$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$=$ R.S.

11. L.S. $= \frac{\sin\alpha}{\cos\alpha}\cdot\sqrt{\cos^2\alpha}$

$= \frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha$

$= \sin\alpha$

$=$ R.S.

12. L.S. $= \frac{1}{\sin\alpha}\cdot\sqrt{\cos^2\alpha}$

$= \frac{\cos\alpha}{\sin\alpha}$

$= \tan\alpha$

$=$ R.S.

13. L.S. $= \sec^2 A\cdot\cos^2 A$

$= \frac{1}{\cos^2 A}\cdot\cos^2 A$

$= 1$

$=$ R.S.

14. L.S. $= \tan^2 A\cdot\cot^2 A$

$= \frac{\sin^2 A}{\cos^2 A}\cdot\frac{\cos^2 A}{\sin^2 A}$

$= 1$

$=$ R.S.

15. L.S. $= \sin^2\theta\cdot\sec^2\theta$

$= \sin^2\theta\cdot\left(\frac{1}{\cos^2\theta}\right)$

$= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$=$ R.S.

16. L.S. $= \cos\alpha \cdot \frac{1}{\sin\alpha} \cdot \tan\alpha$

$= \frac{\cos\alpha}{\sin\alpha} \cdot \frac{\sin\alpha}{\cos\alpha}$

$= 1$

$=$ R.S.

17. L.S. $= \sin^2 A \cdot \operatorname{cosec}^2 A$

$= \sin^2 A \cdot \frac{1}{\sin^2 A}$

$= 1$

$=$ R.S.

18. L.S. $= \cot^2 A \cdot \tan^2 A$

$= \frac{\cos^2 A}{\sin^2 A} \cdot \frac{\sin^2 A}{\cos^2 A}$

$= 1$

$=$ R.S.

19. L.S. $= \sin^2 A \cdot \operatorname{cosec}^2 A$

$= \frac{\sin^2 A}{\sin^2 A}$

$= 1$

$=$ R.S.

20. L.S. $= \sin\alpha \cdot \frac{1}{\cos\alpha} \cdot \cot\alpha$

$= \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha}$

$= 1$

$=$ R.S.

21. L.S. $= \cos\alpha \cdot \operatorname{cosec}\alpha$

$= \frac{\cos\alpha}{\sin\alpha}$

$= \cot\alpha$

$= \sqrt{\operatorname{cosec}^2\alpha - 1}$

$=$ R.S.

22. L.S. $= \sin^2\theta \cdot \left(\cot^2\theta + 1\right)$

$= \sin^2\theta \cdot \operatorname{cosec}^2\theta$

$= \frac{\sin^2\theta}{\sin^2\theta}$

$= 1$

$=$ R.S.

23. L.S. $= \sec^2\theta \cdot \cos^2\theta$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$= 1$

$=$ R.S.

24. L.S. $= \frac{\sin^2\theta}{\cos^2\theta}$

$= \tan^2\theta$

$= \sec^2\theta - 1$

$=$ R.S.

25. L.S. $= \frac{1}{\sin^2\theta} \cdot \frac{\sin^2\theta}{\cos^2\theta} - 1$

$= \sec^2\theta - 1$

$= \tan^2\theta$

$=$ R.S.

26. L.S. $= \cos^2 A + \sin^2 A$

$= 1$

$=$ R.S.

27. L.S. $= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$

$= \frac{1 - \sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A}$

$= 1$

$=$ R.S.

28. L.S. $= \sin^2 A + \cos^2 A$

$= 1$

$=$ R.S.

29. L.S. $= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$

$= \frac{1 - \sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A}$

$= 1$

$=$ R.S.

30. L.S. $= \left(\sin^2\alpha-\cos^2\alpha\right)\left(\sin^2\alpha+\cos^2\alpha\right)$

$= \sin^2\alpha-\cos^2\alpha$

$= \left(1-\cos^2\alpha\right)-\cos^2\alpha$

$= 1-2\cos^2\alpha$

$=$ R.S.

31. L.S. $= \left(\sec^2\alpha-1\right)\left(\sec^2\alpha+1\right)$

$= \tan^2\alpha\cdot\left(\sec^2\alpha-1+2\right)$

$= \tan^2\alpha\cdot\left(\tan^2\alpha+2\right)$

$= 2\tan^2\alpha+\tan^4\alpha$

$=$ R.S.

32. L.S. $= \left(\operatorname{cosec}^2\alpha-1\right)\left(\operatorname{cosec}^2\alpha+1\right)$

$= \cot^2\alpha\cdot\left(\operatorname{cosec}^2\alpha-1+2\right)$

$= \cot^2\alpha\cdot\left(\cot^2\alpha+2\right)$

$= 2\cot^2\alpha+\cot^4\alpha$

$=$ R.S.

33. L.S. $= \left(\dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{1}{\sin\alpha}\right)^2-\left(\sin\alpha\cdot\dfrac{1}{\cos\alpha}\right)^2$

$= \left(\dfrac{1}{\cos\alpha}\right)^2-\left(\dfrac{\sin\alpha}{\cos\alpha}\right)^2$

$= \dfrac{1-\sin^2\alpha}{\cos^2\alpha}$

$= \dfrac{\cos^2\alpha}{\cos^2\alpha}$

$= 1$

$=$ R.S.

34. L.S. $= \left(\dfrac{1}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}\right)^2-\left(\cos\theta\cdot\dfrac{1}{\sin\theta}\right)^2$

$= \left(\dfrac{1}{\sin\theta}\right)^2-\left(\dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \dfrac{1-\cos^2\theta}{\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$

$= 1$

$=$ R.S.

35. L.S. $= \dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{\sin^4\theta-\cos^4\theta}{\cos^2\theta\cdot\sin^2\theta}$

$= \dfrac{\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)}{\cos^2\theta\cdot\sin^2\theta}$

$= \dfrac{\sin^2\theta-\cos^2\theta}{\cos^2\theta\cdot\sin^2\theta}$

$= \dfrac{1}{\cos^2\theta}-\dfrac{1}{\sin^2\theta}$

$= \sec^2\theta-\operatorname{cosec}^2\theta$

$=$ R.S.

**P.12**

1. L.S. $= \dfrac{\sin\alpha}{\cos\alpha}\cdot\dfrac{\cos^2\alpha}{\sin^2\alpha}$

$= \dfrac{\cos\alpha}{\sin\alpha}$

$= \dfrac{1}{\tan\alpha}$

$=$ R.S.

2. L.S. $= \dfrac{1}{\cos^2\alpha}\cdot\dfrac{\cos\alpha}{\sin\alpha}\cdot\sin^2\alpha$

$= \dfrac{\sin\alpha}{\cos\alpha}$

$= \tan\alpha$

$=$ R.S.

3. L.S. $= 1-1+\cos\theta$

$= \cos\theta$

$= \sin\theta\cdot\dfrac{\cos\theta}{\sin\theta}$

$= \sin\theta\cot\theta$

$=$ R.S.

4. L.S. $= \left(1-\cos\theta\right)\left(\dfrac{1}{\cos\theta}\right)$

$= \dfrac{1}{\cos\theta}-1$

$= \sec\theta-1$

$=$ R.S.

5. L.S. $= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$

$= \frac{1-\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$

$=$ R.S.

6. L.S, $= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}$

$= \sec\theta\operatorname{cosec}\theta$

$=$ R.S.

7. L.S. $= \operatorname{cosec}A \cdot \tan A \cdot \cos A$

$= \frac{1}{\sin A} \cdot \frac{\sin A}{\cos A} \cdot \cos A$

$= 1$

$=$ R.S.

8. L.S. $= \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta$

$+\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta$

$= 1+1$

$= 2$

$=$ R.S.

9. L.S. $= 1 + 2\tan\theta - \tan^2\theta$

$+1 - 2\tan\theta + 1\tan^2\theta$

$= 2\left(1-\tan^2\theta\right)$

$= 2\sec^2\theta$

$=$ R.S.

10. L.S. $= \cot^2\theta - 2\cot\theta + 1$

$+\cot^2\theta + 2\cot\theta + 1$

$= 2\left(\cot^2\theta + 1\right)$

$= 2\operatorname{cosec}^2\theta$

$=$ R.S.

11. L.S. $= \sin^2 A\left(\operatorname{cosec}^2 A\right) + \cos^2 A\left(\sec^2 A\right)$

$= \frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\cos^2 A}$

$= 1+1$

$= 2$

$=$ R.S.

12. L.S. $= \cos^2 A\left(1+\tan^2 A - \tan^2 A\right)$

$+\sin^2 A\left(1+\cot^2 A - \cot^2 A\right)$

$= \cos^2 A + \sin^2 A$

$= 1$

$=$ R.S.

13. L.S. $= \cot^2\alpha\left(1+\cot^2\alpha\right)$

$= \cot^2\alpha\left(\operatorname{cosec}^2\alpha\right)$

$= \left(\operatorname{cosec}^2\alpha - 1\right)\left(\operatorname{cosec}^2\alpha\right)$

$= \operatorname{cosec}^4\alpha - \operatorname{cosec}^2\alpha$

$=$ R.S.

14. L.S. $= \frac{\sin^2\alpha}{\cos^2\alpha} \cdot \frac{1}{\sec^2\alpha} \cdot \operatorname{cosec}^2\alpha \cdot \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \frac{\sin^2\alpha}{\cos^2\alpha} \cdot \cos^2\alpha \cdot \frac{1}{\sin^2\alpha} \cdot \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \sin^2\alpha\sec^2\alpha$

$=$ R.S.

15. L.S. $= \frac{1+\sin\alpha+1-\sin\alpha}{(1-\sin\alpha)(1+\sin\alpha)}$

$= \frac{2}{1-\sin^2\alpha}$

$= \frac{2}{\cos^2\alpha}$

$= 2\sec^2\alpha$

$=$ R.S.

16. L.S. $= \tan\alpha\cdot\left(\frac{1}{\sec\alpha-1}+\frac{1}{\sec\alpha+1}\right)$

$= \tan\alpha\cdot\left(\frac{\sec\alpha+1+\sec\alpha-1}{\sec^2\alpha-1}\right)$

$= \tan\alpha\cdot\frac{2\sec\alpha}{\tan^2\alpha}$

$= \frac{2\sec\alpha}{\tan\alpha}$

$= \frac{2}{\cos\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}$

$= \frac{2}{\sin\alpha}$

$= 2\operatorname{cosec}\alpha$

$=$ R.S.

17. L.S. $= \frac{1+\operatorname{cosec}^2\alpha+1+\sin^2\alpha}{1+\sin^2\alpha\cdot\operatorname{cosec}^2\alpha+\sin^2\alpha+\operatorname{cosec}^2\alpha}$

$= \frac{2+\sin^2\alpha+\operatorname{cosec}^2\alpha}{2+\sin^2\alpha+\operatorname{cosec}^2\alpha}$

$= 1$

$=$ R.S.

18. L.S. $= \left(\frac{1}{\cos\theta}+\frac{1}{\sin\theta}\right)(\sin\theta+\cos\theta)$

$= \frac{\sin\theta}{\cos\theta}+1+1+\frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}+2$

$= \frac{1}{\cos\theta\sin\theta}+2$

$= \sec\theta\operatorname{cosec}\theta+2$

$=$ R.S.

19. L.S. $= (\cos\theta-\sin\theta)\left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)$

$= (\cos\theta-\sin\theta)\left(\frac{\cos\theta-\sin\theta}{\cos\theta\sin\theta}\right)$

$= \frac{\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta}{\cos\theta\sin\theta}$

$= \frac{1-2\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta}-2$

$= \sec\theta\operatorname{cosec}\theta-2$

$=$ R.S.

20. L.S. $= (1+\cot\theta)^2-(\operatorname{cosec}\theta)^2$

$= 1+2\cot\theta+\cot^2\theta-\operatorname{cosec}^2\theta$

$= 2\cot\theta+\left(1+\cot^2\theta\right)-\operatorname{cosec}^2\theta$

$= 2\cot\theta+\operatorname{cosec}^2\theta-\operatorname{cosec}^2\theta$

$= 2\cot\theta$

$=$ R.S.

21. L.S. $= (\sec\theta)^2-(\tan\theta-1)^2$

$= \sec^2\theta-\left(\tan^2\theta-2\tan\theta+1\right)$

$= \sec^2\theta-\left(1+\tan^2\theta\right)+2\tan\theta$

$= \sec^2\theta-\sec^2\theta+2\tan\theta$

$= 2\tan\theta$

$=$ R.S.

22. L.S. $= \sin^2 A+2+\operatorname{cosec}^2 A$
$+\cos^2 A+2+\sec^2 A$

$= \left(\sin^2 A+\cos^2 A\right)+2+2$
$+\left(\operatorname{cosec}^2 A-1\right)+1$
$+\left(\sec^2 A-1\right)+1$

$= 1+2+2+\cot^2 A+\tan^2 A+2$

$= \tan^2 A+\cot^2 A+7$

$=$ R.S.

23. L.S. $= \sec^2 A\operatorname{cosec}^2 A+\sec^2 A\cot^2 A$
$+\tan^2 A\operatorname{cosec}^2 A+1$

$= \sec^2 A\operatorname{cosec}^2 A+\frac{1}{\cos^2 A}\cdot\frac{\cos^2 A}{\sin^2 A}$
$+\frac{\sin^2 A}{\cos^2 A}\cdot\frac{1}{\sin^2 A}+1$

$= 1+\sec^2 A\operatorname{cosec}^2 A+\frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A}$

$= 1+2\sec^2 A\operatorname{cosec}^2 A$

$=$ R.S.

24. L.S. $= 1+\sin^2 A+\cos^2 A$

$+2\cos A-2\sin A-2\sin A\cos A$

$= 1+1+2(\cos A-\sin A-\sin A\cos A)$

$= 2(\cos A(1-\sin A)+(1-\sin A))$

$= 2(1-\sin A)(1+\cos A)$

$=$ R.S.

25. L.S. $= \sin A+\dfrac{\sin^2 A}{\cos A}+\cos A+\dfrac{\cos^2 A}{\sin A}$

$= \left(\sin A+\dfrac{\cos^2 A}{\sin A}\right)+\left(\cos A+\dfrac{\sin^2 A}{\cos A}\right)$

$= \dfrac{\sin^2 A+\cos^2 A}{\sin A}+\dfrac{\cos^2 A+\sin^2 A}{\cos A}$

$= \dfrac{1}{\sin A}+\dfrac{1}{\cos A}$

$= \operatorname{cosec} A+\sec A$

$=$ R.S.

26. L.S. $= (\sin\theta+2\cos\theta)(2\tan\theta+1)$

$= \dfrac{2\sin^2\theta}{\cos\theta}+\sin\theta+4\sin\theta+2\cos\theta$

$= \dfrac{2\sin^2\theta+2\cos^2\theta}{\cos\theta}+5\sin\theta$

$= \dfrac{2}{\cos\theta}+5\sin\theta$

$= 2\sec\theta+5\sin\theta$

$=$ R.S.

27. L.S. $= \left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)^2$

$= \left(\dfrac{1+\sin\theta}{\cos\theta}\right)^2$

$= \dfrac{(1+\sin\theta)^2}{1-\sin^2\theta}$

$= \dfrac{(1+\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{1+\sin\theta}{1-\sin\theta}$

$=$ R.S.

28. L.S. $= \dfrac{\cos\theta(2\sin\theta-1)}{(1-\cos^2\theta)-\sin\theta+\sin^2\theta}$

$= \dfrac{\cos\theta(2\sin\theta-1)}{2\sin^2\theta-\sin\theta}$

$= \dfrac{\cos\theta(2\sin\theta-1)}{\sin\theta(2\sin\theta-1)}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$=$ R.S.

29. L.S. $= \dfrac{\cos^2\theta}{\sin^2\theta}\left(\dfrac{\sec\theta-1}{1+\sin\theta}\right)+\dfrac{1}{\cos^2\theta}\left(\dfrac{\sin\theta-1}{1+\sec\theta}\right)$

$= \dfrac{\cos^4\theta(\sec^2\theta-1)+\sin^2\theta(\sin^2\theta-1)}{\sin^2\theta(1+\sin\theta)\cos^2\theta(1+\sec\theta)}$

$= \dfrac{\cos^4\theta(\tan^2\theta)+\sin^2\theta(-\cos^2\theta)}{\sin^2\theta(1+\sin\theta)\cos^2\theta(1+\sec\theta)}$

$= \dfrac{\cos^2\theta\sin^2\theta-\cos^2\theta\sin^2\theta}{\sin^2\theta(1+\sin\theta)\cos^2\theta(1+\sec\theta)}$

$= 0$

$=$ R.S.

30. L.S. $= \dfrac{\sin^2\alpha}{\cos^2\alpha}+\dfrac{1}{\cos^2\beta}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha}+\dfrac{\sin^2\beta+\cos^2\beta}{\cos^2\beta}$

$= \dfrac{\sin^2\alpha\cos^2\beta+\sin^2\beta\cos^2\alpha+\cos^2\alpha\cos^2\beta}{\cos^2\alpha\cos^2\beta}$

$= \dfrac{\cos^2\beta(\sin^2\alpha+\cos^2\alpha)+\sin^2\beta\cos^2\alpha}{\cos^2\alpha\cos^2\beta}$

$= \dfrac{\cos^2\beta}{\cos^2\alpha\cos^2\beta}+\dfrac{\sin^2\beta\cos^2\alpha}{\cos^2\alpha\cos^2\beta}$

$= \dfrac{1}{\cos^2\alpha}+\dfrac{\sin^2\beta}{\cos^2\beta}$

$= \sec^2\alpha+\tan^2\beta$

$=$ R.S.

31. L.S. $= \dfrac{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\cos\beta}{\sin\beta}}{\dfrac{\cos\alpha}{\sin\alpha}+\dfrac{\sin\beta}{\cos\beta}}$

$= \dfrac{\sin\alpha\sin\beta+\cos\alpha\cos\beta}{\cos\alpha\sin\beta}\cdot\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}$

$= \dfrac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta}$

$= \dfrac{\tan\alpha}{\tan\beta}$

$=$ R.S.

32. L.S. $= \dfrac{\dfrac{\sin\alpha}{\cos\alpha}-\dfrac{\cos\beta}{\sin\beta}}{\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\sin\beta}{\cos\beta}}$

$= \dfrac{\sin\alpha\sin\beta-\cos\alpha\cos\beta}{\cos\alpha\sin\beta}\cdot\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$

$= -\dfrac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta}$

$= -\dfrac{\cot\beta}{\cot\alpha}$

$=$ R.S.

33. L.S. $= \dfrac{\tan\beta}{\tan\alpha}\left(\tan\alpha+\dfrac{1}{\tan\beta}\right)$

$= \tan\beta+\dfrac{1}{\tan\alpha}$

$= \cot\alpha+\tan\beta$

$=$ R.S.

34. L.S. $= \sin^2\alpha\cdot\left(1-\sin^2\beta\right)-\left(1-\sin^2\alpha\right)\cdot\sin^2\beta$

$= \sin^2\alpha-\sin^2\alpha\sin^2\beta$

$-\sin^2\beta+\sin^2\alpha\sin^2\beta$

$= \sin^2\alpha-\sin^2\beta$

$=$ R.S.

35. L.S. $= \left(1+\tan^2\alpha\right)\cdot\tan^2\beta-\tan^2\alpha\left(1+\tan^2\beta\right)$

$= \tan^2\beta+\tan^2\alpha\tan^2\beta$

$-\tan^2\alpha-\tan^2\alpha\tan^2\beta$

$= \tan^2\beta-\tan^2\alpha$

$=$ R.S.

36. L.S. $= \sin^2\alpha\cos^2\beta+2\sin\alpha\cos\beta\cos\alpha\sin\beta$

$+\cos^2\alpha\sin^2\beta+\cos^2\alpha\cos^2\beta$

$-2\cos\alpha\cos\beta\sin\alpha\sin\beta+\sin^2\alpha\sin^2\beta$

$= \sin^2\alpha\cos^2\beta+\cos^2\alpha\cos^2\beta$

$+\cos^2\alpha\sin^2\beta+\sin^2\alpha\sin^2\beta$

$= \left(\sin^2\alpha+\cos^2\alpha\right)\cos^2\beta$

$+\left(\cos^2\alpha+\sin^2\alpha\right)\sin^2\beta$

$= \cos^2\beta+\sin^2\beta$

$= 1$

$=$ R.S.

หรือใช้ความรู้ฟังก์ชันตรีโกณมิติของผลบวกมุม

L.S. $= \sin^2(\alpha+\beta)+\cos^2(\alpha+\beta)$

$= 1$

$=$ R.S.

**P.17**

1. L.S. $= \cos A \cdot \tan A$

$= \cos A \cdot \dfrac{\sin A}{\cos A}$

$= \sin A$

$=$ R.S.

2. L.S. $= \sin A \cdot \cot A \cdot \operatorname{cosec} A$

$= \sin A \cdot \cot A \cdot \dfrac{1}{\sin A}$

$= \cot A$

$=$ R.S.

3. L.S. $= \cos A \cdot \tan A \cdot \cot A \cdot \sec A$

$= \cos A \cdot \dfrac{\sin A}{\cos A} \cdot \dfrac{\cos A}{\sin A} \cdot \dfrac{1}{\cos A}$

$= 1$

$=$ R.S.

4. L.S. $= \sin A \cdot \sin A + \cos A \cdot \cos A$

$= \sin^2 A + \cos^2 A$

$= 1$

$=$ R.S.

5. L.S. $= \sin A \cdot \sec A$

$= \sin A \cdot \dfrac{1}{\cos A}$

$= \tan A$

$=$ R.S.

6. L.S. $= \sec^2 A$

$= 1 + \tan^2 A$

$= 1 + \dfrac{\sin^2 A}{\cos^2 A}$

$= 1 + \sin^2 A \sec^2 A$

$= 1 + \sin^2 A \operatorname{cosec}^2 (90° - A)$

$=$ R.S.

7. L.S. $= \sin A \cdot \cot A \cdot \tan A \cdot \operatorname{cosec} A$

$= \sin A \cdot \dfrac{\cos A}{\sin A} \cdot \dfrac{\sin A}{\cos A} \cdot \dfrac{1}{\sin A}$

$= 1$

$=$ R.S.

8. L.S. $= \operatorname{cosec} A - \cot A \cdot \sin A \cdot \cot A$

$= \dfrac{1}{\sin A} - \dfrac{\cos A}{\sin A} \cdot \sin A \cdot \dfrac{\cos A}{\sin A}$

$= \dfrac{1}{\sin A} - \dfrac{\cos^2 A}{\sin A}$

$= \dfrac{1 - \cos^2 A}{\sin A}$

$= \dfrac{\sin^2 A}{\sin A}$

$= \sin A$

$=$ R.S.

9. L.S. $= \tan^2 A \operatorname{cosec}^2 A - \sin^2 A \sec^2 A$

$= \dfrac{\sin^2 A}{\cos^2 A} \cdot \dfrac{1}{\sin^2 A} - \sin^2 A \cdot \dfrac{1}{\cos^2 A}$

$= \dfrac{1}{\cos^2 A} - \dfrac{\sin^2 A}{\cos^2 A}$

$= \dfrac{1 - \sin^2 A}{\cos^2 A}$

$= \dfrac{\cos^2 A}{\cos^2 A}$

$= 1$

$=$ R.S.

10. L.S. $= \cot A + \tan A$

$= \dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}$

$= \dfrac{\cos^2 A + \sin^2 A}{\sin A \cos A}$

$= \dfrac{1}{\sin A \cos A}$

$= \operatorname{cosec} A \cdot \sec A$

$= \operatorname{cosec} A \operatorname{cosec} (90° - A)$

$=$ R.S.

11. L.S. $= \dfrac{\cos A}{\operatorname{cosec} A} \cdot \dfrac{\cot A}{\cos A}$

$= \cos A \cdot \sin A \cdot \dfrac{\cos A}{\sin A} \cdot \dfrac{1}{\cos A}$

$= \cos A$

$=$ R.S.

12. L.S. $= \dfrac{\csc^2 A \tan^2 A}{\tan A} \cdot \dfrac{\cot A}{\sec^2 A}$

$= \csc^2 A \cdot \tan A \cdot \dfrac{\cot A}{\sec^2 A}$

$= \dfrac{1}{\sin^2 A} \cdot \dfrac{\sin A}{\cos A} \cdot \dfrac{\cos A}{\sin A} \cdot \cos^2 A$

$= \dfrac{\cos^2 A}{\sin^2 A}$

$=$ R.S.

13. L.S. $= \dfrac{\tan A}{\csc^2 A} \cdot \dfrac{\sec A \cot^3 A}{\cos^2 A}$

$= \dfrac{\sin A}{\cos A} \cdot \sin^2 A \cdot \dfrac{1}{\cos A} \cdot \dfrac{\cos^3 A}{\sin^3 A} \cdot \dfrac{1}{\cos^2 A}$

$= \dfrac{1}{\cos A}$

$= \sec A$

$= \sqrt{\tan^2 A + 1}$

$=$ R.S.

14. L.S. $= \dfrac{\sin^2 A}{1 - \cos A}$

$= \dfrac{1 - \cos^2 A}{1 - \cos A}$

$= \dfrac{(1 - \cos A)(1 + \cos A)}{1 - \cos A}$

$= 1 + \cos A$

$= 1 + \sin(90° - A)$

$=$ R.S.

15. L.S. $= \dfrac{\cot^2 A - \cos^2 A}{\cot A + \cos A}$

$= \dfrac{(\cot A + \cos A)(\cot A - \cos A)}{\cot A + \cos A}$

$= \cot A - \cos A$

$= \tan(90° - A) - \cos A$

$=$ R.S.

16. $x \cdot \cos A \cdot \dfrac{\cos A}{\sin A} = \sin A$

$x = \dfrac{\sin^2 A}{\cos^2 A}$

$\therefore \quad x = \tan^2 A$

17. $\sec A \sec A - x \tan A = 1$

$\sec^2 A - 1 = x \tan A$

$\tan^2 A = x \tan A$

$\therefore \quad x = \tan A$

## P.57

1. L.S. $= \dfrac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}$

$= \dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}$

$= \dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}$

$= \tan A + \tan B$

$=$ R.S.

2. L.S. $= (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B)$

$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$

$= \cos^2 A(1 - \sin^2 B) - (1 - \cos^2 A)(\sin^2 B)$

$= \cos^2 A - \cos^2 A \sin^2 B - \sin^2 B + \cos^2 A \sin^2 B$

$= \cos^2 A - \sin^2 B$

$=$ R.S.

3. L.S. $= (\sin A \cos B + \cos A \sin A)(\sin A \cos B - \cos A \sin B)$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= (1 - \cos^2 A)(\cos^2 B) - (\cos^2 A)(1 - \cos^2 B)$

$= \cos^2 B - \cos^2 B \cos^2 A - \cos^2 A + \cos^2 A \cos^2 B$

$= \cos^2 B - \cos^2 A$

$=$ R.S.

4. L.S. $= (\cos 45° \cos\theta - \sin 45° \sin\theta)$

$-(\sin 45° \cos\theta - \cos 45° \sin\theta)$

$= \dfrac{\cos\theta}{\sqrt{2}} - \dfrac{\sin\theta}{\sqrt{2}} - \dfrac{\cos\theta}{\sqrt{2}} + \dfrac{\sin\theta}{\sqrt{2}}$

$= 0$

$=$ R.S.

5. L.S. $= \sin(4\theta + \theta)$

$= \sin(5\theta)$

$= \sin(3\theta + 2\theta)$

$= \sin 3\theta \cos 2\theta + \cos 3\theta \sin 2\theta$

$=$ R.S.

**P.66**

1. L.S. $= \dfrac{2\sin A\cos A}{1+2\cos^2 A-1}$

$= \dfrac{2\sin A\cos A}{2\cos^2 A}$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$

$=$ R.S.

2. L.S. $= \dfrac{2\sin A\cos A}{1-(1-2\sin^2 A)}$

$= \dfrac{2\sin A\cos A}{2\sin^2 A}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A$

$=$ R.S.

3. L.S. $= \dfrac{1-(1-2\sin^2\frac{A}{2})}{2\sin\frac{A}{2}\cos\frac{A}{2}}$

$= \dfrac{2\sin^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}$

$= \dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$

$= \tan\dfrac{A}{2}$

$=$ R.S.

4. L.S. $= \dfrac{2\sin\frac{A}{2}\cos\frac{A}{2}}{1+2\cos^2\frac{A}{2}-1}$

$= \dfrac{2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos^2\frac{A}{2}}$

$= \dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$

$= \tan\dfrac{A}{2}$

$=$ R.S.

5. L.S. $= \dfrac{\frac{\cos A}{\sin A}-\frac{\sin A}{\cos A}}{\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}}$

$= \dfrac{\cos^2 A-\sin^2 A}{\frac{\sin A\cos A}{\cos^2 A+\sin^2 A}}$

$= \dfrac{\cos^2 A-\sin^2 A}{\sin A\cos A}\cdot\dfrac{\sin A\cos A}{1}$

$= \cos^2 A-\sin^2 A$

$= \cos 2A$

$=$ R.S.

6. L.S. $= \dfrac{\frac{1}{\cos A}-1}{\frac{1}{\cos A}}$

$= \dfrac{1-\cos A}{\cos A}\cdot\cos A$

$= 1-(1-2\sin^2\dfrac{A}{2})$

$= 2\sin^2\dfrac{A}{2}$

$=$ R.S.

7. L.S. $= \dfrac{\tan 45^\circ+\tan A}{1-\tan 45^\circ\tan A}-\dfrac{\tan 45^\circ-\tan A}{1+\tan 45^\circ\tan A}$

$= \dfrac{1+\tan A}{1-\tan A}-\dfrac{1-\tan A}{1+\tan A}$

$= \dfrac{(1+2\tan A+\tan^2 A)-(1-2\tan A+\tan^2 A)}{1-\tan^2 A}$

$= \dfrac{4\tan A}{1-\tan^2 A}$

$= 2\cdot\dfrac{2\tan A}{1-\tan^2 A}$

$= 2\tan 2A$

$=$ R.S.

8. L.S. $= 2\cos^2 2A-1$

$= 2(2\cos^2 A-1)^2-1$

$= 2(4\cos^4 A-4\cos^2 A+1)-1$

$= 8\cos^4 A-8\cos^2 A+2-1$

$= 8\cos^4 A-8\cos^2 A+1$

$=$ R.S.

9. L.S. $= \dfrac{\left(\sin\frac{A}{2}-\cos\frac{A}{2}\right)\left(\sin^2\frac{A}{2}+\sin\frac{A}{2}\cos\frac{A}{2}+\cos^2\frac{A}{2}\right)}{\sin\frac{A}{2}-\cos\frac{A}{2}}$

$= \sin^2\frac{A}{2}+\sin\frac{A}{2}\cos\frac{A}{2}+\cos^2\frac{A}{2}$

$= \sin^2\frac{A}{2}+\cos^2\frac{A}{2}+\sin\frac{A}{2}\cos\frac{A}{2}$

$= 1+\sin\frac{A}{2}\cos\frac{A}{2}$

$= \dfrac{2+2\sin\frac{A}{2}\cos\frac{A}{2}}{2}$

$= \dfrac{2+\sin A}{2}$

$=$ R.S.

10. L.S. $= \dfrac{\cos a}{\sin a}-\dfrac{\sin a}{\cos a}$

$= \dfrac{\cos^2 a-\sin^2 a}{\sin a\cos a}$

$= \dfrac{2(\cos^2 a-\sin^2 a)}{2\sin a\cos a}$

$= \dfrac{2\cos 2a}{\sin 2a}$

$= 2\cot a$

$=$ R.S.

11. L.S. $= \sin^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}+\cos^2\frac{A}{2}$

$= \sin^2\frac{A}{2}+\cos^2\frac{A}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}$

$= 1+\sin A$

$=$ R.S.

12. L.S. $= \sin^2\frac{A}{2}-2\sin\frac{A}{2}\cos\frac{A}{2}+\cos^2\frac{A}{2}$

$= \sin^2\frac{A}{2}+\cos^2\frac{A}{2}-2\sin\frac{A}{2}\cos\frac{A}{2}$

$= 1-\sin A$

$=$ R.S.

13. L.S. $= (\cos^2 A+\sin^2 A)(\cos^4 A-\cos^2 A\sin^2 A+\sin^4 A)$

$= (1)(\cos^4 A+2\cos^2 A\sin^2 A+\sin^4 A-3\cos^2 A\sin^2 A)$

$= (\cos^2 A+\sin^2 A)^2-\frac{3}{4}(4\cos^2 A\sin^2 A)$

$= (1)-\frac{3}{4}(2\cos A\sin A)^2$

$= 1-\frac{3}{4}(2\sin A\cos A)^2$

$= 1-\frac{3}{4}\sin^2 2A$

$=$ R.S.

14. L.S. $= (\cos^2 A-\sin^2 A)(\cos^4 A+\cos^2 A\sin^2 A+\sin^2 A)$

$= (\cos 2A)((\cos^2 A+\sin^2 A)^2-\cos^2 A\sin^2 A)$

$= \cos 2A(1-\frac{1}{4}(4\cos^2 A\sin^2 A)$

$= \cos 2A(1-\frac{1}{4}(2\sin A\cos A)^2)$

$= \cos 2A(1-\frac{1}{4}\sin^2 2A)$

$=$ R.S.

15. L.S. $= \dfrac{1}{\cos A}-\dfrac{\sin A}{\cos A}$

$= \dfrac{1-\sin A}{\cos A}$

$= \dfrac{1-\dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}}{\dfrac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}}}$

$= \dfrac{1+\tan^2\frac{A}{2}-2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}\cdot\dfrac{1+\tan^2\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$= \dfrac{1-2\tan\frac{A}{2}+\tan^2\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$= \dfrac{(1-\tan\frac{A}{2})^2}{(1-\tan\frac{A}{2})(1+\tan\frac{A}{2})}$

$= \dfrac{1-\tan\frac{A}{2}}{1+\tan\frac{A}{2}}$

$= \dfrac{1-\tan\frac{A}{2}}{1+(1)\tan\frac{A}{2}}$

$= \dfrac{\tan 45^\circ-\tan\frac{A}{2}}{1+\tan 45^\circ\tan\frac{A}{2}}$

$= \tan(45^\circ-\frac{A}{2})$

$=$ R.S.

**P.69**

1. L.S. $= \dfrac{3\cos A+(4\cos^3 A-3\cos A)}{3\sin A-(3\sin A-4\sin^3 A)}$

$= \dfrac{3\cos A+4\cos^3 A-3\cos A}{3\sin A-3\sin A+4\sin^3 A}$

$= \dfrac{4\cos^3 A}{4\sin^3 A}$

$= \cot^3 A$

$=$ R.S.

2. L.S. $= \dfrac{\cos^3 A-(4\cos^3 A-3\cos A)}{\cos A}$

$= \dfrac{\cos^3 A-4\cos^3 A+3\cos A}{\cos A}$

$= \dfrac{3\cos A-3\cos^3 A}{\cos A}$

$= \dfrac{3\cos A(1-\cos^2 A)}{\cos A}$

$= 3(1-\cos^2 A)$

$= 3\sin^2 A$

$=$ R.S.

3. L.S. $= 4\sin^3 A(4\cos^3 A-3\cos A)$

$+4\cos^3 A(3\sin A-4\sin^3 A)$

$= 16\sin^3 A\cos^3 A-12\sin^3 A\cos A$

$+12\cos^3 A\sin A-16\sin^3 A$

$= 12\cos^3 A\sin A-12\sin^3 A\cos A$

$= 6(2\sin A\cos A)(\cos^2 A-\sin^2 A)$

$= 6(\sin 2A)(\cos 2A)$

$= 3(2\sin 2A\cos 2A)$

$= 3\sin 4A$

$=$ R.S.

4. จาก $\tan(3A) = \tan(A+2A)$

$\tan(3A) = \dfrac{\tan 2A+\tan A}{1-\tan 2A\tan A}$

$\tan 3A-\tan 3A\tan 2A\tan A = \tan 2A+\tan A$

$\therefore \tan 3A-\tan 2A-\tan A = \tan 3A\tan 2A\tan A$

5. L.S. $= \dfrac{4\cos^3 A-3\cos A-3\sin A+4\sin^3 A}{\cos A+\sin A}$

$= \dfrac{4(\cos^3 A+\sin^3 A)-3(\cos A+\sin A)}{\cos A+\sin A}$

$= \dfrac{4(\cos^3 A+\sin^3 A)}{\cos A+\sin A}-3$

$= \dfrac{4(\cos A+\sin A)(\cos^2 A-\cos A\sin A+\sin^2 A)}{\cos A+\sin A}-3$

$= 4(\cos^2 A-\cos A\sin A+\sin^2 A)-3$

$= 4(\cos^2 A+\sin^2 A-\frac{1}{2}(2\sin A\cos A))-3$

$= 4(1-\frac{1}{2}\sin 2A)-3$

$= 4-2\sin 2A-3$

$= 1-2\sin 2A$

$=$ R.S.

**P.74**

1. L.S. $= \cos 3A-(\sin 4A-\sin 2A)$

$= \cos 3A-2\cos 3A\sin A$

$= \cos 3A(1-2\sin A)$

$=$ R.S.

2. L.S. $= \cos 2\theta+(\cos 5\theta+\cos\theta)$

$= \cos 2\theta+2\cos 3\theta\cos 2\theta$

$= \cos 2\theta(1+2\cos 3\theta)$

$=$ R.S.

3. L.S. $= (\cos 3A+\cos 15A)+(\cos 5A+\cos 7A)$

$= 2\cos 9A\cos 6A+2\cos 6A\cos A$

$= 2(\cos 9A+\cos A)\cos 6A$

$= 2(2\cos 5A\cos 4A)\cos 6A$

$= 4\cos 4A\cos 5A\cos 6A$

$=$ R.S.

4. L.S. $= \dfrac{(\sin\alpha+\sin 5\alpha)+(\sin 2\alpha+\sin 4\alpha)}{(\cos\alpha+\cos 5\alpha)+(\cos 2\alpha+\cos 4\alpha)}$

$= \dfrac{2\sin 3\alpha\cos 2\alpha+2\sin 3\alpha\cos\alpha}{2\cos 3\alpha\cos 2\alpha+2\cos 3\alpha\cos\alpha}$

$= \dfrac{\sin 3\alpha(2\cos 2\alpha+2\cos\alpha)}{\cos 3\alpha(2\cos 2\alpha+2\cos\alpha)}$

$= \tan 3\alpha$

$=$ R.S.

5. L.S. $= \dfrac{2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})}{2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})}$

$= \dfrac{\sin(\frac{A+B}{2})}{\cos(\frac{A+B}{2})} \cdot \dfrac{\cos(\frac{A-B}{2})}{\sin(\frac{A-B}{2})}$

$= \tan(\frac{A+B}{2})\cot(\frac{A-B}{2})$

$=$ R.S.

**P.81**

1. L.S. $= \dfrac{1-\cos 2A}{2} + \dfrac{1-\cos 2B}{2} - \dfrac{1-\cos 2C}{2}$

$= \dfrac{1-(\cos 2A+\cos 2B)+\cos 2C}{2}$

$= \dfrac{1-2\cos(A+B)\cos(A-B)+(2\cos^2 C-1)}{2}$

$= \dfrac{-2\cos(A+B)\cos(A-B)+2\cos^2 C}{2}$

$= \dfrac{-2(-\cos C)\cos(A-B)+2\cos^2 C}{2}$

$= \cos C\cos(A-B)+\cos^2 C$

$= \cos C(\cos(A-B)+\cos C)$

$= \cos C(\cos(A-B)-\cos(A+B))$

$= \cos C(-2\sin A\sin(-B))$

$= \cos C(-2\sin A(-\sin B))$

$= 2\sin A\sin B\cos C$

$=$ R.S.

2. L.S. $= (\sin 2A+\sin 2C)-\sin 2B$

$= 2\sin(A+C)\sin(A-C)-2\sin B\cos B$

$= 2\sin B\cos(A-C)-2\sin B\cos B$

$= 2\sin B(\cos(A-C)-\cos B)$

$= 2\sin B(\cos(A-C)-(-\cos(A+C))$

$= 2\sin B(\cos(A-C)+\cos(A+C))$

$= 2\sin B(2\cos A\cos(-C))$

$= 2\sin B(2\cos A\cos C)$

$= 4\cos A\sin B\cos C$

$=$ R.S.

3. L.S. $= \sin 2A-(\sin 2B+\sin 2C)$

$= 2\sin A\cos A-2\sin(B+C)\cos(B-C)$

$= 2\sin A\cos A-2\sin A\cos(B-C)$

$= 2\sin A(\cos A-\cos(B-C))$

$= 2\sin A(-\cos(B+C)-\cos(B-C))$

$= -2\sin A(\cos(B+C)+\cos(B-C))$

$= -2\sin A(2\cos B\cos(-C))$

$= -2\sin A(2\cos B\cos C)$

$= -4\sin A\cos B\cos C$

$=$ R.S.

4. L.S. $= 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2} - 2\sin\dfrac{C}{2}\cos\dfrac{C}{2}$

$= 2\cos\dfrac{C}{2}\cos\dfrac{A-B}{2} - 2\sin\dfrac{C}{2}\cos\dfrac{C}{2}$

$= 2\cos\dfrac{C}{2}(\cos\dfrac{A-B}{2}-\sin\dfrac{C}{2})$

$= 2\cos\dfrac{C}{2}(\cos\dfrac{A-B}{2}-\cos\dfrac{A+B}{2})$

$= 2\cos\dfrac{C}{2}(-2\sin\dfrac{A}{2}\sin(-\dfrac{B}{2}))$

$= 2\cos\dfrac{C}{2}(-2\sin\dfrac{A}{2}(-\sin\dfrac{B}{2}))$

$= 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$

$=$ R.S.

5. L.S. $= (\cos A+\cos C)-\cos B$

$= 2\cos\dfrac{A+C}{2}\cos\dfrac{A-C}{2} - (1-2\sin^2\dfrac{B}{2})$

$= 2\sin\dfrac{A+C}{2}\cos\dfrac{A-C}{2} - 1 + 2\sin^2\dfrac{B}{2}$

$= 2\sin\dfrac{B}{2}\cos\dfrac{A-C}{2} + 2\sin^2\dfrac{B}{2} - 1$

$= 2\sin\dfrac{B}{2}(\cos\dfrac{A-C}{2}+\sin\dfrac{B}{2}) - 1$

$= 2\sin\dfrac{B}{2}(\cos\dfrac{A-C}{2}+\cos\dfrac{A+C}{2}) - 1$

$= 2\sin\dfrac{B}{2}(2\cos\dfrac{A}{2}\cos(-\dfrac{C}{2})) - 1$

$= 2\sin\dfrac{B}{2}(2\cos\dfrac{A}{2}\cos\dfrac{C}{2}) - 1$

$= 4\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2} - 1$

$=$ R.S.

6. L.S. $= \dfrac{(\sin B+\sin C)-\sin A}{(\sin B+\sin C)+\sin A}$

$= \dfrac{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}-2\sin\frac{A}{2}\cos\frac{A}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$

$= \dfrac{2\cos\frac{A}{2}\cos\frac{B-C}{2}-2\sin\frac{A}{2}\cos\frac{A}{2}}{2\cos\frac{A}{2}\cos\frac{B-C}{2}+2\sin\frac{A}{2}\cos\frac{A}{2}}$

$= \dfrac{2\cos\frac{A}{2}(\cos\frac{B-C}{2}-\sin\frac{A}{2})}{2\cos\frac{A}{2}(\cos\frac{B-C}{2}+\sin\frac{A}{2})}$

$= \dfrac{\cos\frac{B-C}{2}-\sin\frac{A}{2}}{\cos\frac{B-C}{2}+\sin\frac{A}{2}}$

$= \dfrac{\cos\frac{B-C}{2}-\cos\frac{B+C}{2}}{\cos\frac{B-C}{2}+\cos\frac{B+C}{2}}$

$= \dfrac{-2\sin\frac{B}{2}\sin(-\frac{C}{2})}{2\cos\frac{B}{2}\cos(-\frac{C}{2})}$

$= \dfrac{-2\sin\frac{B}{2}(-\sin\frac{C}{2})}{2\cos\frac{B}{2}\cos\frac{C}{2}}$

$= \dfrac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}$

$= \tan\frac{B}{2}\tan\frac{C}{2}$

$=$ R.S.

7. L.S. $= \frac{1-\cos 2A}{2}+\frac{1-\cos 2B}{2}+\frac{1-\cos 2C}{2}+2\cos A\cos B\cos C$

$= \frac{3+\cos 2A+\cos 2B+\cos 2C}{2}+2\cos A\cos B\cos C$

$= \frac{3+2\cos^2 A-1+2\cos(B+C)\cos(B-C)}{2}+2\cos A\cos B\cos C$

$= \frac{2+2\cos^2 A+2\cos(B+C)\cos(B-C)}{2}+2\cos A\cos B\cos C$

$= 1+\cos^2 A+\cos(B+C)\cos(B-C)+2\cos A\cos B\cos C$

$= 1+\cos^2 A-\cos A\cos(B-C)+2\cos A\cos B\cos C$

$= 1+\cos A(\cos A-\cos(B-C))+2\cos A\cos B\cos C$

$= 1+\cos A(-\cos(B+C)-\cos(B-C)+2\cos A\cos B\cos C$

$= 1+\cos A(-2\cos B\cos C)+2\cos A\cos B\cos C$

$= 1-2\cos A\cos B\cos C+2\cos A\cos B\cos C$

$= 1$

$=$ R.S.

8. L.S. $= \dfrac{1-\cos A}{2}+\dfrac{1-\cos B}{2}+\dfrac{1-\cos C}{2}$

$= \dfrac{3-\cos A-(\cos B+\cos C)}{2}$

$= \dfrac{3-(1-2\sin^2\frac{A}{2})-(2\cos\frac{B+C}{2}\cos\frac{B-C}{2})}{2}$

$= \dfrac{2+2\sin^2\frac{A}{2}-2\cos\frac{B+C}{2}\cos\frac{B-C}{2}}{2}$

$= 1+\sin^2\frac{A}{2}-\cos\frac{B+C}{2}\cos\frac{B-C}{2}$

$= 1+\sin^2\frac{A}{2}-\sin\frac{A}{2}\cos\frac{B-C}{2}$

$= 1+\sin\frac{A}{2}(\sin\frac{A}{2}-\cos\frac{B-C}{2})$

$= 1+\sin\frac{A}{2}(\cos\frac{B+C}{2}-\cos\frac{B-C}{2})$

$= 1+\sin\frac{A}{2}(-2\sin\frac{B}{2}\sin\frac{C}{2})$

$= 1-2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$=$ R.S.